

Ms Cooper is teaching about fractions to a Year 6 class (aged 10-11). Each pupil holds a rectangular piece of paper and a pen. Ms Cooper asks the pupils to split their pieces of paper into two equal parts. Most pupils draw a line through the centre of the rectangle, parallel to one of the sides, either vertically or horizontally (Figure 1). Asked what makes the two parts equal, most pupils fold their papers showing to Ms Cooper that the two smaller pieces of paper coincide exactly.

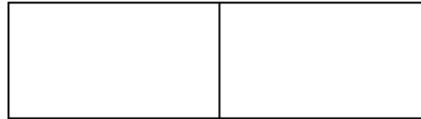
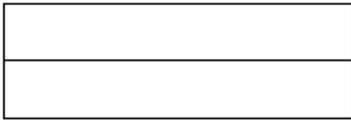


Figure 1. A rectangle split in two (horizontally and vertically)

Ms Cooper then notices that one pupil, Ellie, has drawn a diagonal line instead (Figure 2).

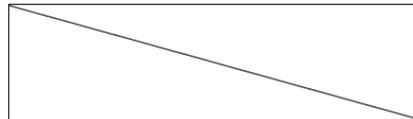


Figure 2. Ellie's rectangle split in two (diagonally).

She praises Ellie for her originality and asks her whether the two parts are equal. Ellie cuts the paper along the diagonal and places the two pieces on top of each other showing Ms Cooper that they coincide exactly.

Ms Cooper then asks the class to split their pieces of paper into four equal parts. Again, most pupils draw two lines parallel to one of the sides (Figure 3) and fold their paper to show that the four parts coincide exactly.

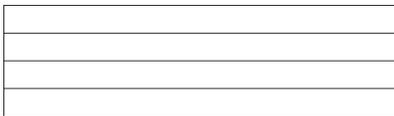


Figure 3. A rectangle split in four (horizontally and vertically)

Ellie, again, draws a second diagonal line instead (Figure 4).

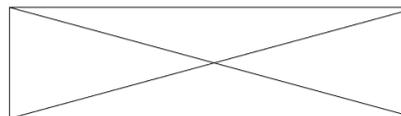


Figure 4. Ellie's rectangle split in four (diagonally).

At seeing Ellie's solution, another student, Neil, protests:

**Neil:** These don't look the same to me. If that was a brownie and I was given this bit [shows the triangle on the left], I would not be happy! The class laughs and many pupils' nod. Ms Cooper then asks Ellie whether she sees the four parts as equal. Ellie replies as follows:

**Ellie:** Well, they are not the same *shape*... two of them are kind of short and fat [she shows the top and bottom triangles] and the other two are thinner and taller [she shows the triangles on the left and right]. But I have a feeling they are the same *size*. Maybe? Hold on! I have an idea...

She draws two dotted lines, one vertically and one horizontally (Figure 5) and colours in the two new, smaller triangles, one red and the other blue.

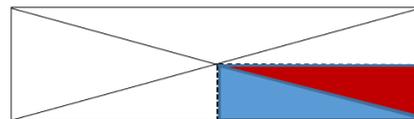


Figure 5. Ellie's rectangle split in four (diagonally).

She then cuts the two-coloured triangles, places one on top of the other and jumps with excitement:

**Ellie:** Look! These two, the red and the blue, *are* the same! And – and! – every one of my four large triangles, the top, the bottom, the left and the right are made up of the two small ones. Yes! Not the same shape, but the same size! Happy with your brownie piece now, Neil?

A third pupil, Barack, then jumps in:

**Barack:** I think Ellie maybe right. But, I don't trust all this cutting and colouring... Isn't there a formula for ...what's it called... area for each one of these shapes? My sister is doing her GCSE and she was showing me the other day the formula for the area of a rectangle. I think it's one side times the other side or something. What's the one for triangles? Maybe we can use that to make sure Ellie is right...

**Neil:** ...or wrong! You are both such show-offs! These four bits [points at Fig. 4] don't look the same. End of story!

1. Is Ellie, right? Explain your answer.
2. How would you respond to Ellie?
3. How would you respond to Barack?
4. How would you respond to Neil?
5. How would you respond to the whole class and conclude the lesson?

\* Based on activities for the 2018-19 portfolio of learning outcomes designed by Elena Nardi ([e.nardi@uea.ac.uk](mailto:e.nardi@uea.ac.uk)) for use in UEA's BA Education Year 3 module *Children, teachers and mathematics: Changing public discourses about mathematics*. Inspired by the "Jason" episode (p.20-21) in: Rowland, T. (2013). The knowledge quartet: the genesis and application of a framework for analysing mathematics teaching and deepening teachers' mathematics knowledge. *Journal of Education* 1(3), 15-43.