Which Fraction is Smaller?*

Below is an episode from a lesson in the Y6 class of Ms Chambers.

Ms Chambers asks the students to work on the following problem:

Which fraction is smaller, $\frac{3}{4}$ *or* $\frac{5}{6}$*?*

One of the students, **Neil**, complains that this is too complicated and that he never got fractions: 'they are no use anyway!', he proclaims. Ms Chambers invites views from the class on Neil's complaint. Anna raises her hand and Ms Chambers invites her to speak.

Anna: I don't think this is as complicated as Neil thinks. But I do think you are tricking us! I drew this (Fig. 1) and this (Fig. 2). There is one box left in both and I think this means that none of these is smaller than the other. They are equal!

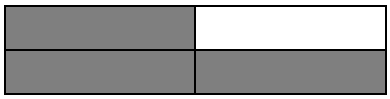

Figure 1.

Figure 2.

Another student, Barack, then asks permission to speak.

Barack: Why do you have to make these drawings? We have a perfectly good method for this: $\frac{3}{4}$ is the same as $\frac{9}{12}$

[*he writes:* $\frac{3}{4} = \frac{3x3}{4x3} = \frac{9}{12}$] and $\frac{5}{6}$ is the same as $\frac{10}{12}$ [*he writes:* $\frac{5}{6} = \frac{5x2}{6x2} = \frac{10}{12}$]

$\frac{10}{12}$ is obviously bigger than $\frac{9}{12}$ so $\frac{5}{6}$ is definitely bigger than $\frac{3}{4}$.

A third student, Clive, waves his hand impatiently. Ms Chambers signals to him that he can speak.

Clive: What a waste of time! Why do you have to make these drawings *[addressing Anna]*? And why do you have to put us through all these… [*pointing contemptuously at Barack's calculations].* These two are equal because when I add 2 to 3 and then 2 to 4, I get $\frac{3+2}{4+2}$ which is (yeah!) $\frac{5}{6}$! Problem solved!

'Thank you, all', says Ms Chambers, 'Quite a few ideas! Shall we take them one by one?'

1. Solve the mathematical problem (Which fraction is smaller, $\frac{3}{4}$ *or* $\frac{5}{6}$) in the above episode. Explain your answer.
2. How would you respond to Anna?
3. How would you respond to Barack?
4. How would you respond to Clive?
5. How would you conclude the lesson in a way that provides a satisfactory response to the mathematical problem and appeases Neil's exasperated comment?

* Based on activities and examination questions designed by Elena Nardi (e.nardi@uea.ac.uk) for use in UEA's BA Education Year 3 module *Children, teachers and mathematics: Changing public discourses about mathematics* between 2012 and 2016.